C/SiC HX Thermal and Mechanical Analysis

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ABSTRACT

This report summarizes UCB’s initial results and methods on C/SiC heat exchanger thermal and mechanical analysis for high temperature heat transfer under support from DOE National Hydrogen Initiative program (NHI) through UNLV HTHX project, as a part of efforts for nuclear hydrogen production. The intermediate heat exchanger (IHX) transfers heat from high temperature high pressure helium to a liquid salt intermediate loop which couples to hydrogen production loops. The IHXs operates at temperature from 600°C to 1000°C and pressure difference up to 8 MPa. The operating conditions are extremely challenging for metals to function in, while it is not challenging for carbon and silicon carbide composites. To be economically viable for hydrogen production, ceramic heat exchangers must have costs under a few tens of dollars per thermal kilowatt. Plate-type ceramic heat exchanger with relatively small flow channels provide a good candidate approach for this purpose, because they can obtain high power densities and low mechanical stresses with thin heat transfer surfaces and small amounts of material. Thermal and stress analysis are carried out to identify optimal designs for core heat transfer unit, inlet and outlet manifolds, distribution channels, and channel dividers. Finding detailed stress distribution in a complete heat exchanger with direct FEM (Finite Element Method) requires an order of billion FEM computation units and million hours PC computing time. Therefore, it is not practical to analyze the entire heat exchanger design directly. We propose an alternative method to obtain approximate stresses that only requires several days to finish in a fast PC. The methods are composed of three steps. First, the heat exchanger is broken down into several regions. Unit cell models are built based on each region that captures all of the most important features of that region. The effective mechanical and thermal properties for each unit cell are then founded through FEM simulations. Second, average stress distribution in an overall model composed of various unit cell regions is computed by using the effective mechanical and thermal properties. Third, these average stress values are then applied to the unit cells to find localized points of high stresses. Pro/Mechanica module (Pro/M) in the Pro/E Wildfire Edition is used for FEM stress analysis.
1. INTRODUCTION

Nuclear hydrogen production requires an IHX to transfer high temperature heat from high temperature gas cooled reactor to an intermediate loop which couples with hydrogen production loops. The IHXs operates at temperature from 600°C to 1000°C and pressure difference up to 8 MPa. The operating conditions are extremely challenging for metals to function in, while it is not challenging for carbon and silicon carbide composites. LSI and PI carbon-carbon composites can maintain nearly full mechanical strength at high temperatures (up to 1400°C), have low residual porosity and are compatible with molten salt and high-pressure helium [1]. These materials are relatively simple to fabricate and have relatively low cost. To be economically viable for hydrogen production, ceramic heat exchangers must have costs under a few tens of dollars per thermal kilowatt. Plate-type ceramic heat exchanger with relatively small flow channels provide a good candidate approach for this purpose, because they can obtain high power densities and low mechanical stresses with thin heat transfer surfaces and small amounts of material.

To fabricate compact plate-type heat exchangers, one side of each plate is die-embossed or milled, to provide appropriate flow channels, leaving behind fins or ribs that would provide enhanced heat transfer, as well as the mechanical connection to the smooth side of the next plate. For green carbon-carbon material, milling can be performed readily with standard numerically controlled milling machines. Alternatively, plates can be molded with flow channels, as has been demonstrated for carbon-carbon composite plates fabricated at ORNL for fuel cells [2]. For assembly, the ends of the fins and other remaining unmachined surfaces around the machined flow channels would be coated with phenolic adhesive, the plate stack assembled, header pipes bonded and reinforced, and the resulting monolith pyrolysed under compression. Then liquid silicon or polymers would be infiltrated to reaction bond the plates and headers together, forming a compact heat exchanger monolith.

Figure 1 illustrates discontinuous fin geometry for molten salt-to-helium compact heat exchangers. The cross-sectional area of the fins and the thickness of the remaining plate below the machined channels would be adjusted to provide sufficient strength to resist thermal and mechanical stresses. For the case in which the heat exchanger is immersed into a helium environment, detailed unit cell stress analysis as shown in Figure 2 has indicated that the stresses are dominantly compressive and can be accommodated with relative ease. Figure 3 shows a preliminary draft plate design for the compact offset fin plate heat exchangers. Helium plates and molten salt plates are alternatively joined together to form a heat exchanger module. This design is applicable to compact counterflow heat exchanger, where there is a substantial difference between the volumetric flow rates of the two fluids. Applications can include liquid-to-gas heat exchangers and gas-to-gas heat exchangers where there exists a large difference in the volumetric flow rates. The helium side plate has several equal spacing flow dividers to reduce flow maldistribution. Earlier preliminary mechanical stress analysis shows that the stress on the helium divider in the distribution region is very low. Most of it comes from the compressive force of high pressure helium on the divider itself. Global mechanical stress analysis and thermal stress analysis are carried out in this study.
**Figure 1**: Cut away view through a plate showing alternating molten salt (top and bottom arrows) and helium (middle arrows) flow channels. Dark bands at the top of each fin indicate the location of reaction-bonded joints between each plate.

**Figure 2**: Stress and temperature distributions in a plate type LSI composite heat exchanger.
Figure 3: Plates design for compact offset fin plate heat exchanger.

CAD design and analysis software Pro/Engineer (Pro/E) is used for 3-D design, thermal and mechanical analyses of the offset fin compact plate heat exchangers. Pro/E has the advantage of combining design and analysis work together. Pro/Mechanica module (Pro/M) in the Pro/E Wildfire Edition is used for stress analysis. Pro/M uses p-type finite elements (FEM) to compute its solution [3]. One of the key advantages of p-type finite elements is that they allow solution adaptivity without requiring mesh refinement. With standard “h-type” finite elements, once a solution is obtained, the only way to improve its quality consists of repeating the calculation using a finer mesh - this process is well known to be time consuming, complex and problematic in several ways. In contrast, with p-type elements, the maximum polynomial orders of basis functions used to approximate the solution can be increased locally as needed. The solution process can then be repeated on the same mesh, with the new increased polynomial orders. Such an adaptivity step (often called a pass in Pro/M) can be repeated, if desired, to achieve even greater accuracy. In the p-type finite element method, basis functions are constructed in a way that the maximum polynomial order of approximation can be selected independently for each edge, face, and solid in the mesh. Pro/M selects polynomial orders independently for each edge, and the polynomial order of approximation on each face and solid is then selected based on the choices made for the underlying edges. Therefore, the goal of a p-order adaptivity method consists of selecting appropriate polynomial orders for each edge that will lead to an overall solution providing good quality results using acceptable elapsed time and computing resource.
Although finite element analysis is capable of finding accurate results over various kinds of geometries, for complex geometries the amount of time needed to process finite-element models can become prohibitively large. As the number of polygons and the order of equations fitted increase, the number of equations that must be solved increases exponentially. For example, finding the stresses accurate to 10% on a 600-polygon unit cell model can take up to 30 minutes on the fastest personal computer. One unit cell in the core heat transfer region has length scale about 1 cm. The total length scale for a compact heat exchanger is 1 m. Entire heat exchanger needs $6 \cdot 10^8$ polygons. Finding detailed stress distribution in a complete heat exchanger with direct FEM (Finite Element Method) requires an order of million hours PC computing time. Therefore, it is not practical to analyze the entire heat exchanger design directly. We propose an alternative approach that gives slightly less accurate results but is more versatile and far less time consuming. The following part describes the basic ideas.

The compact plate fin heat exchanger is composed of many alternating Molten Salt (MS) and Helium (He) plates. For stress analysis, each pair of heat exchanger plates is basically identical. So only one pair of plates (one MS plate and one He plate) is needed for the purpose of stress analysis. To analyze the steady-state thermal and mechanical stresses in a full-size high temperature heat exchanger module, one pair of plates in the heat exchanger is broken down into multiple similar regions. Unit cell models are built based on each region that captures all of the most important features of that region. Figure 4 shows the various regions that are grouped together because of similar features. Figure 5 shows pictures of the unit cell models built from the regions defined in figure 4. In order from left to right, top to bottom, they are unit cells A, B, C, and D. These models are called unit cells because when one reflects each cell across its six boundary surfaces one can recreate the geometry found in the corresponding region. In other words, the unit cells all have cubic symmetry. Note that one can only create unit cells out of regions that have symmetrical patterns. Certain approximations need to be applied when a region cannot be recreated from symmetrical unit cells. For example, unit cell A has round fillets on the inner edge in its top portion but no rounds in the bottom. Yet this cell will give approximately the right answer because the rounds represent a relatively small mass compared to the main body of the cell. This and other approximations employed are pointed out in appropriate locations of this report.
Figure 4: Division of unit cell model regions in a plate.

Figure 5: Unit cell models.
The methods we use to obtain approximate thermal and mechanical stresses are composed of three steps. First, the effective mechanical and thermal properties for each unit cell are founded through various FEM simulations; Second, average stress distribution in an overall model composed of various unit cell regions is computed by using the effective mechanical and thermal properties; Third, these average stress values are then applied to the unit cells to find localized points of high stresses. For example, if the result of average stress analysis shows that region A has a maximum compressive stress of -15 MPa (all compressive stresses are written as negative numbers), an overall pressure of 15 MPa is applied across all six surfaces of unit cell A. The maximum compressive stress found in the unit cell A in this FEM simulation would be the max actual local compressive stress in region A. This also pinpoints locations where stresses are greatest. Similar method is employed to find the maximum tensile stress.

2. EFFECTIVE MECHANICAL AND THERMAL PROPERTIES FOR UNIT CELLS

2.1 Normal Stress Moduli

Assumptions

A unit cell is orthotropic, which means it deforms in different ways along each of its three axes (3 Young’s moduli, 3 Poisson ratios, 3 Shear moduli). The material that made up the unit cell is uniform and isotropic. Helium pressure is 10 MPa and MS pressure is 0.1 MPa.

The Models

Four models are used to find all relevant normal stress moduli, one for each unit cell. The model for unit cell A is used here as an example to show how the unit cell models are created. Unit cell A is set up to include one salt channel and one helium channel, one salt fin and one helium fin. The model is created by having a symmetry plane (normal to z-axis) cuts a fin in half at its waist, and another symmetry plane (normal to x-axis) cuts the staggered fin along the middle. The third symmetry plane, which is normal to the y-axis, cuts in the middle of the MS and HE plate. Despite the root of the fins being rounded and the top being un-rounded, the two sides of the third symmetry plane are approximately the same. This is one of the unit cell approximations previously mentioned in the procedure. Unit cells B, C, and D were created on the same principles as A, any differences and approximations will be noted in Appendix A.

Constraints

The process of assigning constraints is described here using unit cell A as an example. On each face of the unit cell, a rigid flat plane is imposed. To make sure it was rigid and flat, translation in the direction of corresponding axis is fixed, while rotations in the other 2 axes are fixed. The other 3 degrees of freedom are freed. The constraint surface normal to the x-axis on the positive x side is called X1, the constraint surface parallel to
X1 but is on the negative x side is called X2. Similarly, the other 4 constraint surfaces are called Y1, Y2, Z1, and Z2. Initially, the constraint surfaces are set at zero displacement so an initial test will show the stresses on the unit cell with no displacement. Since the moduli are found by calculating change in stress versus change in strain, it is important to know what the stress is at zero displacement. Subsequent constraint sets vary the displacements to obtain the moduli.

**Loads**

The loads used in the model are pressure loads representing the fluid pressure present in each channel. For the MS channels, 0.1 MPa is used, while 10 MPa is used for He channel surfaces.

**Properties of the heat exchanger materials**

Table 1 shows material properties of typical SiSiC composite, which is a potential material for the compact heat exchangers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus [GPa]</td>
<td>300</td>
</tr>
<tr>
<td>Poisson Ratio, [-]</td>
<td>0.2</td>
</tr>
<tr>
<td>Tensile Strength [MPa]</td>
<td>~300</td>
</tr>
<tr>
<td>Coefficient of Thermal Expansion (300 - 1000 °C) [10^-6 /K]</td>
<td>4.5</td>
</tr>
<tr>
<td>Heat Conductivity [W/m·K]</td>
<td>40</td>
</tr>
<tr>
<td>Density [g/cm^3]</td>
<td>3.1</td>
</tr>
<tr>
<td>Specific Heat Capacity [kJ/kg·K]</td>
<td>1.1</td>
</tr>
</tbody>
</table>

**Methods**

The whole process is performed using Pro/M software. Three variables are set up to track the reactions on the symmetry boundaries (which coincide with the constraint surfaces). The first variable tracks reaction in the x-direction on the X1 constraint surface, the second variable y-direction reaction, and etc. For each unit cell, four stress tests were run in Pro/M. The initial test has zero displacement on all constraint surfaces. The other three tests were run with the strain described in table 2. The actual displacement set for each test is equal to strain times the length of the unit cell in the corresponding direction. For example, unit cell A is 5 mm long so for test 1, the x displacement for the X1 constraint surface is set to -1.5e-3 mm. The negative displacement indicates simulated compression. The reported reactions are added to corresponding channel forces on each face and divided by the total area on that face to obtain the average stress on that constraint surface. Several matrix calculations were carried out to obtain all nine normal stress moduli.
Matrix Calculations

Based on the results of the four tests performed on each unit cell, the nine normal stress moduli (Young’s moduli and poison ratios) can be extracted. The relation between stress and strain is as follows:

\[ \varepsilon = D \times \sigma, \]  

(1)

Where \( \varepsilon \) is the strain tensor, \( \sigma \) the stress tensor. The inverse elastic tensor (\( D \)) for orthotropic material is defined as [Reddy, 1999]:

<table>
<thead>
<tr>
<th></th>
<th>1/E_x</th>
<th>- v_xy/E_x</th>
<th>- v_xz/E_x</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>- v_yx/E_y</td>
<td>1/E_y</td>
<td>- v_yz/E_y</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>- v_zx/E_z</td>
<td>- v_zy/E_z</td>
<td>1/E_z</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0</th>
<th>1/G_xy</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/G_yz</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/G_zx</td>
<td></td>
</tr>
</tbody>
</table>

Where \( E \) is normal stress Young’s modulus, \( G \) shear stress modulus, and \( \nu \) Poisson’s ratio. For orthotropic material, the stress-strain relation can be rewritten as six equations. If we only look at the top three equations (the ones that do not involve shear stress), we see nine variables and three equations. However, if three tests are performed and each test generates three independent sets of stress and their corresponding strains, nine equations with nine variables are formed. Simultaneously solving these nine equations yield the nine variables (the nine normal stress moduli). The zero-displacement test is needed because the stress-strain relation only works with changes in stresses and changes in strains, so a reference point is needed to calculate those changes. Detail derivation for the equations used in matrix calculations is attached in Appendix B.

Results

The unit cell A is taken as an example to show how to find nine moduli. Table 3 shows the surface areas of the constraint surfaces and the lengths of the unit cell A in the corresponding axes.
Table 3. Geometry data for unit cell A

<table>
<thead>
<tr>
<th></th>
<th>Solid Surface Area (mm$^2$)</th>
<th>MS Channel Area (mm$^2$)</th>
<th>He Channel Area (mm$^2$)</th>
<th>Total Area (mm$^2$)</th>
<th>Length of Cell Along That Axis (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>37.26</td>
<td>6.07</td>
<td>12.17</td>
<td>55.5</td>
<td>3</td>
</tr>
<tr>
<td>Y1</td>
<td>14.52</td>
<td>18.78</td>
<td>0</td>
<td>33.3</td>
<td>5</td>
</tr>
<tr>
<td>Z1</td>
<td>9.12</td>
<td>1.44</td>
<td>4.44</td>
<td>15</td>
<td>11.1</td>
</tr>
</tbody>
</table>

The equation to calculate average stress on each constraint surface is,

$$\sigma_{\text{ave}} = \frac{(R_s - \text{PMS} \times \text{AMS} - \text{PHe} \times \text{AHe})}{A_{\text{total}}} \quad (2)$$

Where $P$ is pressure, $A$ surface area, $R_s$ the reaction at corresponding solid face as reported by Pro/M. The pressure forces are negative because they were forces of the fluid in surrounding cells. Table 4 shows the results of the four Pro/M stress tests for the unit cell A.

Table 4. Stress results of Pro/M tests for the unit cell A

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{X1}$</th>
<th>$\sigma_{Y1}$</th>
<th>$\sigma_{Z1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-displacement</td>
<td>-4.606</td>
<td>-5.903</td>
<td>-4.153</td>
</tr>
<tr>
<td>Test 1</td>
<td>-69.476</td>
<td>-50.408</td>
<td>-106.530</td>
</tr>
<tr>
<td>Test 2</td>
<td>-79.009</td>
<td>-56.125</td>
<td>-76.750</td>
</tr>
<tr>
<td>Test 3</td>
<td>-92.479</td>
<td>-44.882</td>
<td>-93.716</td>
</tr>
</tbody>
</table>

From the results and based on the equations in Appendix B, the nine effective normal stress moduli were calculated as follows,

$$E_{\text{eff},x} = 1.310 \times 10^{11} \text{ Pa}$$
$$E_{\text{eff},y} = 7.110 \times 10^{10} \text{ Pa}$$
$$E_{\text{eff},z} = 1.634 \times 10^{11} \text{ Pa}$$

Poisson Ratios were as follows,

$$\nu_{xy} = 0.113$$
$$\nu_{xz} = 0.199$$
$$\nu_{yx} = 0.213$$
$$\nu_{yz} = 0.192$$
$$\nu_{zx} = 0.157$$
$$\nu_{zy} = 0.078$$

To fully define an orthotropic cell, only three independent Poisson ratios are needed. $\nu_{xy}, \nu_{xz},$ and $\nu_{yz}$ can be calculated from $\nu_{yx}, \nu_{zx},$ and $\nu_{zy}$. The Poisson ratios in the $yx$, $zx$, and $zy$ directions are the ones asked for by Pro/M when entering material properties so those are the ones that will be used for this method.

In addition to effective moduli, each unit cell has an effective density that must be taken into account. This number can be easily calculated as a fraction of the density of the original material based on the volume occupied by the unit cell. Equation 3 can be used.

$$\rho_{\text{eff}} = \rho \cdot (V_s / V_{\text{total}}) \quad (3)$$
Where $\rho_{\text{eff}}$ is effective density of the unit cell, $\rho$ density of the original material, $V_s$ the solid volume of the unit cell and $V_{\text{total}}$ the total volume of the unit cell.

Table 5 summarizes the results of the normal stress test for all four unit cells.

<table>
<thead>
<tr>
<th>Unit Cell</th>
<th>$E_{\text{eff},x}$ [Pa]</th>
<th>$E_{\text{eff},y}$ [Pa]</th>
<th>$E_{\text{eff},z}$ [Pa]</th>
<th>$\nu_{yx}$</th>
<th>$\nu_{zx}$</th>
<th>$\nu_{zy}$</th>
<th>$\rho_{\text{eff}}$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.310e11</td>
<td>7.11e10</td>
<td>1.634e11</td>
<td>0.2128</td>
<td>0.1575</td>
<td>0.0785</td>
<td>1810</td>
</tr>
<tr>
<td>B</td>
<td>1.683e11</td>
<td>8.30e10</td>
<td>1.758e11</td>
<td>0.1937</td>
<td>0.1882</td>
<td>0.0812</td>
<td>2049</td>
</tr>
<tr>
<td>C</td>
<td>1.599e11</td>
<td>5.29e10</td>
<td>1.557e11</td>
<td>0.1848</td>
<td>0.1790</td>
<td>0.0685</td>
<td>1835</td>
</tr>
<tr>
<td>D</td>
<td>2.702e11</td>
<td>2.103e11</td>
<td>2.499e11</td>
<td>0.2014</td>
<td>0.2002</td>
<td>0.1670</td>
<td>2792</td>
</tr>
</tbody>
</table>

Summary

The requirement for a transversely isotropic unit cell is that two of its Young’s moduli and two of its Poisson ratios must be approximately the same; furthermore, the two Poisson ratios must be in the direction of one of the two equal Young’s moduli and the direction of the third unequal Young’s moduli. Based on table 5, unit cell A is obviously orthotropic as all three of its Young’s moduli are different. However, B, C, and D seem to be transversely isotropic since two of their Young’s moduli and two of their Poisson ratios are equal. But the wrong pairs of Poisson ratios are equal with respect to the pairs of Young’s moduli that are equal. So these three unit cells must also be orthotropic.

2.2 Shear Stress Moduli

Assumptions

The unit cell is orthotropic. The material that made up the unit cell is uniform and isotropic.

Methods

The main difference between the shear stress tests and the normal stress tests is that lateral strain, instead of axial strain, is imposed on the constraint surfaces. For example, for the test to find $G_{xy}$, a displacement in the y-direction is imposed on the X1 face and the X2 face is constrained in the y-direction; both surfaces are constrained in the x-direction. Y1 and Y2 would be left free, and Z1 and Z2 would be constrained in the z-direction. The rotation constraints are set up to allow each surface to rotate in the direction of their corresponding axes. And the Y surfaces could additionally rotate in the z-direction. Figure 6 shows how the shear stress test for $G_{xy}$ on the unit cell A was done.
This setup violates the symmetry constraints. It is unfortunate that it seems to be no way to keep a flat rigid slanted surface boundary on the Y surfaces in Pro/M. To get good results, it is necessary to approximate the shear moduli. Instead of performing the shear test on a single unit cell, the test was performed on a large, almost square-shaped block composed of many unit cells. Two tests were performed for each of the three shear moduli. The results from the xy and the yx test would be averaged to find the $G_{xy}$ moduli. (In orthotropic materials, $G_{xy} = G_{yx}$, the other four shear moduli are paired up in a similar fashion) By convention, Pro/M accepts $G_{xy}$, $G_{xz}$, and $G_{yz}$, so those are the notations that will be used for the three shear moduli. Figure 7 is an example of the model used to find shear moduli on the unit cell A. The more unit cells used to make up the shear test model the more closely the results of each pair of shear test would agree. However, increasing the size of the model significantly increases the calculations time so the models used were around sizes that took about an hour to analyze per run.

Since the strains were set and the reaction forces were reported by Pro/M, the shear moduli could be found with Equation 4,

$$G = \frac{\tau}{\gamma}$$  \hspace{1cm} (4) 

where $G$ is the shear moduli, $\tau$ the shear stress, and $\gamma$ the physical strain. The physical strain is obtained by Equation 5,

$$\gamma_{xy} = \arctan\left(\frac{\text{disp}_{X1-Y}}{L_x}\right)$$ \hspace{1cm} (5) 

where $\text{disp}_{X1-Y}$ is the displacement of constraint surface X1 in the y-direction and $L_x$ is the length of the unit cell in the x-direction. For further reference, engineering shear strain ($\varepsilon_{xy}$) is equal to half of physical strain.
Figure 7: Shear test model for $G_{xy}$ on the unit cell A.

Results

Table 6 summarizes the results of the shear stress tests.

<table>
<thead>
<tr>
<th>Unit cell</th>
<th>$G_{xy}$ [Pa]</th>
<th>$G_{xz}$ [Pa]</th>
<th>$G_{yz}$ [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.89e10</td>
<td>5.99e10</td>
<td>4.18e10</td>
</tr>
<tr>
<td>B</td>
<td>3.03e10</td>
<td>6.61e10</td>
<td>2.34e10</td>
</tr>
<tr>
<td>C</td>
<td>1.85e10</td>
<td>6.81e10</td>
<td>2.87e10</td>
</tr>
<tr>
<td>D</td>
<td>6.69e10</td>
<td>7.35e10</td>
<td>5.02e10</td>
</tr>
</tbody>
</table>

2.3 Thermal Properties

Introduction

Once the mechanical properties of each of the four unit cells are determined, the thermal properties of those cells must be evaluated. The three important thermal properties for
thermal stress analysis are thermal expansion coefficient $\alpha$, specific heat capacity $c_p$, and effective thermal conductivity $k$.

Methods

For orthotropic unit cells, there can be up to three expansion coefficients, one specific heat capacity, and three conduction coefficients. However, our unit cells are not real orthotropic materials; they are effectively orthotropic. No matter what shape the unit cells take on, they would expand the same relative amount in all directions with temperature changes much like an isotropic cell. The three expansion coefficients are equal to the expansion coefficient of the original material regardless of unit cell geometry. An easy way to visualize this is to imagine a cubic unit cell filled with an isotropic solid material and another cubic unit cell that is made from bars of the same material lining all twelve edges. Simple math or common sense can tell that under the same temperature change, the solid cube will expand just as much as the hollow cube frame will. Geometries of the cube and the frame will not influence the effective expansion coefficient of each unit cell.

Since specific heat capacity is a measure of how much energy is stored per mass per temperature change, unit cell geometry once again plays no role. This is why there is only one specific heat capacity for orthotropic unit cells.

On the other hand, there are three conduction coefficients for our unit cells, and these coefficients do depend on geometry. Due to limitations in the ability of Pro/M to output thermal flux data, these coefficients have to be estimated. They are estimated by taking a fraction of the original material’s thermal conductivity based on smallest cross sectional area. For example, for the unit cell A in the x-direction, most of the heat energy is conducted along the plates and not the fins. The plates have a cross-sectional area of 22.2 mm$^2$ in the x-direction, and the total cross-sectional area in the x-direction is 55.5 mm$^2$. The estimated thermal conduction coefficient in the x-direction is equal to 40 W/(m·K) (coefficient value of the original material) times (22.2 / 55.5), which is equal to 16 W/(m·K).

Results

Table 7 summarizes all thermal coefficients. $\alpha$ stands for expansion coefficient, $k$ stands for conduction coefficient, and $c_p$ stands for heat capacity. With these coefficients, the characterization of the entire heat exchanger is complete.

<table>
<thead>
<tr>
<th>Unit Cell</th>
<th>$\alpha$ [1/K]</th>
<th>$k_x$, $k_y$, $k_z$ [W/(m·K)]</th>
<th>$c_p$ [kJ/(kg·K)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4.5e-6</td>
<td>16, 8.9, 16</td>
<td>1.1</td>
</tr>
<tr>
<td>B</td>
<td>4.5e-6</td>
<td>20, 12, 24</td>
<td>1.1</td>
</tr>
<tr>
<td>C</td>
<td>4.5e-6</td>
<td>20, 8.3, 19</td>
<td>1.1</td>
</tr>
<tr>
<td>D</td>
<td>4.5e-6</td>
<td>36, 19, 32</td>
<td>1.1</td>
</tr>
</tbody>
</table>
3. OVERALL STEADY-STATE STRESS ANALYSES

Just as a recap, figure 8 shows the overall heat exchanger plate assembly that describes the original heat exchanger plates. Each region is color coded, and the gray regions represent the original material (marked as region O). The figure also shows the constraints that are applied to the overall model. Since the heat exchanger may be free to expand and contract in real-life, we should use constraints that constrict the free expansion of the model as little as possible. The Y2 surfaces of the model are restrained from moving in the y-direction. (Recall that the Y1 face refers to the surface perpendicular to the y-axis in the positive y-direction, and the Y2 face refers to the surface in the negative y-direction.) Then either the hot molten salt inlet or the cold outlet is restrained in the r and θ directions. These constraints mimic what would happen if either the hot inlet or cold outlet pipe connected to the model are rigid. Results from these two sets of constraints will be presented separately.

Since the heat exchanger would be submerged in a high pressure helium environment, the exposed sides of the heat exchanger are assumed to subject to 10 MPa pressures. The inlet and outlet holes are assumed to subject to pressures of approximately 1 MPa coming from dense molten salt. The Y2 face had been made rigidly flat, but to allow for expansion, the Y1 face was given full freedom. To simulate the pressure from the helium environment in the y-direction we applied a pressure on the Y1 face that effectively mimicked environmental pressure from helium. By applying a pressure of 10.62 MPa, the average pressure across the entire Y1 face, counting the contribution from the inlet/outlet hole, would be equal to 10 MPa.

The thermal load on the overall assembly is based on a simple linear temperature distribution analysis. That is, the temperature on the MS inlet/outlet and the helium inlet/outlet are defined and they are assumed to vary linearly across the entire heat exchanger plate. This assumption is very basic but the results provide some indication of what kind of stresses the real heat exchanger will experience. Taking a simple thermal design for AHTR power conversion heater (MS to He heat exchanger) by UCB as an example, the temperatures on the various inlet and outlet are defined as follows. MS inlet is set at 925°C, MS outlet is set at 860°C, He inlet is set at 622°C, and He outlet is set at 900°C. The heaters for AHTR power conversion system works in more challenging environment than a NGNP IHX (higher pressure difference and higher temperature). Using AHTR heater parameters to study the stresses can give conservative results for NGNP IHX design.

Pro/M requires that we set a reference temperature in order to properly generate thermal loading. The reference temperature will be set dependent upon which hole in the model is being constrained. For the test where the hot inlet is constrained, the reference temperature will be set at 925 °C, and for the cold outlet test, the reference temperature will be set at 860 °C. These temperatures were chosen based on the temperature of the hole being constrained. If one does not pick a reference temperature that is the same as the hole being constrained, artificial (fake) stresses will be generated as the area around the constrained hole tries to contract or expand against a fixed hole. In reality, the hole contracts or expands with the surrounding area.
Figure 8: Overall heat exchanger plate model with constraints.

Figure 9 shows the temperature distribution given by Pro/M based on the previously given temperature constraints. For temperature color scale, white represents hot and blue represents cold. Figure 10 to 13 show, in order, the maximum principle (XP) stresses based on constraining the hot MS inlet, the minimum principle (MP) stresses based on constraining the hot inlet, the XP stresses based on constraining the cold inlet, and the MP stresses based on constraining the cold inlet. Note that analyses of the principle stresses were chosen as oppose to Von Mises stresses because the original material behaves like a ceramic and will fracture if forced to accommodate any significant tensile strain deformation. The phenomenon of fracturing depends on the highest compressive (min principle) and tensile (max principle) stresses present in the material. Also, that the Hot Inlet test refers to the analysis in which the hot MS inlet was constrained; and the Cold Outlet test refers to the analysis in which the cold MS outlet was constrained. Also note that the legends are not the same for every graph; the legends were chosen to highlight important features.
Figure 9: Simple linear temperature distribution.

Figure 10: Max principle stress based on Hot Inlet Test.
Figure 11: Min principle stress based on Hot Inlet Test.

Figure 12: Max principle stress based on Cold Inlet Test.
Recall that the reason one does analyses on the overall heat exchanger plate model is to find the maximum average compressive and tensile stresses present in each of the regions represented by unit cells. However, based on figures 10 to 13, it is apparent that the majority of unit cell A, B, and C are under an average stress very different from the maximum and minimum principle stresses. So to get a better picture and how stresses are distributed in the unit cells, we should take note of the average stresses those regions are under and do localized stress tests with those average values as well as with the max and min values. Table 8 summarizes the max, min, and average stresses in each of the five regions. Max and Min stand for the maximum and the minimum stresses found in the cell. The Avg (XP, MP) columns contain the average max and min principle stresses found in the majority of each cell for each test. And the typical stress values are the average of the averages, representing the typical stress state that the unit cell is in.

### Table 8. Summary of stresses in each unit cell (MPa)

<table>
<thead>
<tr>
<th>Unit Cell</th>
<th>Hot Inlet Test</th>
<th>Cold Outlet Test</th>
<th>Typical Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Min</td>
<td>Avg (XP,MP)</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>-14</td>
<td>-9.5, -10.5</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>-20</td>
<td>-8, -12</td>
</tr>
<tr>
<td>C</td>
<td>50</td>
<td>-23</td>
<td>-7, -12</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>-70</td>
<td>N/A</td>
</tr>
<tr>
<td>O</td>
<td>75</td>
<td>-130</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Figure 13:** Min principle stress based on Cold Inlet Test.
The reason why we will not do a local stress analysis based on average stress values on unit cell D is because the stresses on unit cell D varies greatly across the unit cell, doing an average stress test on unit cell D will not tell us a great deal about the cell. Unit cell O is just the original material, so there is no need to do local stress analysis on unit cell O.

It is interesting to see that the average stress values for the hot and cold tests are nearly identical for unit cell A, B, and C. This suggests that constraining the overall model by the inlet or by the outlet does not significantly affect the average stresses found in the central regions. We would expect this from Saint-Venant’s Principle since the central region is far away from either the inlet or the outlet.

It should be noted that the average stresses felt locally is somewhat different depending on which hole is constrained. It seems that if the hot inlet is constrained, the compressive stress will be higher than the tensile stresses. On the other hand, if the cold outlet is constrained, the tensile stresses increase while the compressive stresses decrease. This can be explained by the temperature distribution calculated earlier. The cold outlet side is contracting while the hot inlet side is expanding. If the hot inlet pipe is rigid, one can expect high compressive stresses as the cold side contracts against the pipe. And if one makes the cold outlet pipe rigid, one can expect tensile stresses from hot side expansion. Since the material in question can resist compressive stress better than tensile stress, making the hot inlet pipe rigid is more desirable than making the cold outlet pipe rigid.

4. LOCAL STEADY-STATE STRESS ANALYSES

After doing the stress analyses on the overall heat exchanger model and looking at general global trends, the results from those analyses are used for local stress tests. For each unit cell, there are at most two maximum average stresses (from the hot and cold global tests), two minimum average stresses, and one typical stress. Fortunately, many of these stress values coincide, which is a testimony to how close the hot and cold global tests agree on regions that are not close to the holes. Unit cell D is the closest to the holes, so the stress values on unit cell D varies the most.

Constraints and Loads

The four models have already been described in section 2 and Appendix A. But for local steady-state stress analyses the same stresses are applied across all six surfaces of each unit cell. Since Pro/M does not allow one to apply a flat constraint surface and apply a pressure on that surface at the same time, one must manipulate the displacement on the constraint surface instead. One can use the tensor matrices obtained in section 2 and multiply them by the desired stress matrix to obtain the strain that has to be applied. Do not forget that it is the change in stress that must be inputted into the stress matrix so one needs to subtract the desired stresses by the zero-displacement stresses. Once one has the needed strain values, one multiplies them by the corresponding unit cell lengths to get the displacement values. The inputted displacement values will generate the correct average stresses on all six surfaces on the unit cell.
Results for Unit Cell A

For simplicity, only the full results for unit cell A will be shown. Key results for the other three unit cells will also be mentioned. Additional results and graphs for unit cells B, C, and D will be shown in Appendix C.

Note that all graphs in this section are shown in a ten-color scale. All regions in red are under stresses higher than the corresponding max value and all regions in dark blue are under stresses lower than the corresponding min value. Regions with the other eight colors are between the min and max values on the scale. For maximum principle stress graphs, the red regions are the high tensile stress zones. The scale is read from the lowest number to the highest number. For minimum principle stress graphs, the blue regions are the high compressive stress zones. The scale is read from the highest number (lowest compressive stress) to the lowest number (highest compressive stress). The lower left corner of each graph contains a set of axes. Red is X, green is Y, and blue is Z. The upper left corner of each graph list, in order, stress type, units, and load set. The information in the upper left corner is not important for our discussion.

Figure 14 and 15 show the maximum principle stresses for unit cell A under the maximum tensile condition of 0 MPa environmental pressures. Environmental pressures refer to the effective pressure felt by all sides of the unit cell based on global stress analyses in section 3.

**Figure 14:** Max principle stress for unit cell A at 0 MPa pressure (scaled at 10 to 80 MPa).
The purpose of Figure 14 is to show the areas that will potentially be under the highest tensile stresses in the heat exchanger plate operating in steady state. The roots of the helium fins are under tensile of stresses of 80 MPa while the fins carry about 35 MPa. Although the highest tensile stress is 129 MPa, this value is found in the inner corner at the top of the helium fin as shown in red lines in Figure 15. Corner geometry is notorious for showing inaccurate stress values under FEM analysis. To get an accurate value, one must take an entire surrounding region and average the stress over the entire region. Since the high tensile stresses in this region drop off rapidly as we travel away from the edge, we can guess that the actual stresses there are on the same magnitude as the stresses found at the root of the helium fin. To check for fracture in this region, a physical model must be made and tested.

Although tensile stresses are a major concern, compressive stresses can also cause the material these tests are based on to fracture. Figure 16 shows the minimum stresses (highest magnitude negative stresses) found in unit cell A under the compressive condition of 15 MPa environmental pressures. Pressures are shown as negative numbers so -20 to -100 MPa actually reads as 20 to 100 MPa compressive stresses. For convenience, compressive stresses will be written as negative numbers and pressures will be written as positive numbers. The minimum principle stress distribution in Figure 16 is very similar to the distribution in Figure 14 except that the molten salt fin is under high compressive stress. The roots of the molten salt fin show stress of up to -80 MPa and the minimum stress in the whole model is -180 MPa. Once again, the minimum stress point is found in an inner corner, except this time it is in the inner corner of the molten salt channel. Stresses in this inner corner can only be found accurately with a physical test.
Figure 16: Min principle stress for unit cell A at 15 MPa pressure (scaled at -20 to -100 MPa).

Note that the conditions in Figure 14 and 16 can only be found in a very small percentage of the regions represented by unit cell A. Recall from table 8 that the majority of region A is under an effective environmental pressure of 10 MPa. Figure 17 and 18 shows the maximum and minimum principles stresses of unit cell A under 10 MPa pressure. As shown in the these figures, the most problematic regions that the majority of unit cell A shown are in the roots of the fins and the inner corner at the top of the fins. The tensile stresses at the roots of the helium fins are 5 MPa, and the compressive stresses at the roots of the molten salt fins are -60 MPa. The overall max tensile stress is 11 MPa and the overall max compressive stress is -95 MPa. These values are well within the strength limits of the original material. Figure 18 also shows a deformed figure with a light blue outline of the original unit cell model. If you look carefully, the high-pressure helium is pushing the plates outward, toward the molten salt channels. This effect is most prominent in between rows of fins where there is the least amount of supporting materials. As a consequence of this effect, the roots of the fins get stretched and that is why those areas become highly concentrated stress zones.

For the inner corners of the unit cell model, we might find that the real stresses there do actually turn out to be as high as Pro/M reports (though it is unlikely). In such event, we should consider a manufacturing process that adds small rounds at those corners. Chemical bonding between plates, for example, should add a small round at those inner corners to help alleviate stress.
Figure 17: Max principle stress for unit cell A at 10 MPa pressure (scaled at -20 to 8 MPa).

Figure 18: Min principle stress for unit cell A at 10 MPa pressure (scaled at -10 to -70 MPa).
Results for Other Unit cells

Table 9 summarizes the results for all five heat exchanger regions and all four unit cells. Note that stresses found at the inner corner are not included in table 9 because they are highly inaccurate. All max/min results are found by analyzing the corresponding unit cells at the maximum and minimum stresses found in the two global analyses. If only the results from the hot test is considered, the compressive stresses will be lower than the values in table 9; if only the results from the cold test is considered, the tensile stresses will be lower. But before examining general trends of the results, first the other unit cells are reviewed.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80</td>
<td>-80</td>
<td>5</td>
<td>-60</td>
</tr>
<tr>
<td>B</td>
<td>90</td>
<td>-150</td>
<td>11</td>
<td>-35</td>
</tr>
<tr>
<td>C</td>
<td>750</td>
<td>-370</td>
<td>10</td>
<td>-35</td>
</tr>
<tr>
<td>D</td>
<td>100</td>
<td>-225</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>O</td>
<td>130</td>
<td>-130</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Figures 19 and 20 show the typical maximum and minimum principle stresses present in unit cell B. A surprising result can be seen in figure 19 where the maximum value does not occur at the inner corners of the molten salt channel. Instead, the maximum value occurs slightly away from the inner corners at the top of the molten salt channels. (Normally, the min/max values occur at the inner corners) This is because the high-pressure helium pushes the top of the molten salt channels downward, making them bulge out. The inner corner is in compression and the top of the channel is in tension from the bulging effect.
**Figure 19:** Max principle stress for the unit cell B at 10 MPa pressure (scaled at -5 to 10 MPa).

**Figure 20:** Min principle stress for unit cell B at 10 MPa pressure (scaled at -10 to -50 MPa).
Figures 21 and 22 show the typical maximum and minimum principle stresses present in unit cell C. Unit cell C exhibits similar stress pattern as unit cell B except typically at a slightly lower compressive stress value. However, as can be seen from table 9, the maximum and minimum stresses found in region C are abnormally high. Recalling from figures 10 to 13, which are the overall heat exchanger test results, there is a corner of region C that is under very high stresses. This high stress translates to abnormally high local stress values in tension and compression. The corner previously mentioned is located to the right of the molten salt outlet hole. If the hole is considered to be a head, the regions of high stresses are located around the neck. This region should be examined closely for stress reduction and future analysis.

![Stress Max Prin (WCS) (N / mm²)](image)

**Figure 21**: Max principle stress for unit cell C at 9.5 MPa pressure (scaled at -20 to 10 MPa).
Figure 22: Min principle stress for unit cell C at 9.5 MPa pressure (scaled at -10 to -50 MPa).

Figures 23 and 24 show the maximum and minimum principle stresses present in unit cell D. Unit cell D is different from the other three unit cells in that it has a lot more solid mass. In this cell, the stresses are much closer to the strength limit than in the other cells. The stresses also vary a lot more across region D than any other region. However, the extra materials also mean more reinforcement against cracking.
Figure 23: Min principle stress for unit cell D at 30 MPa tensile stress (scaled at 40 to -120 MPa).

Figure 24: Max principle stress for unit cell D at 70 MPa Pressure (scaled at –125 MPa to –325 MPa).
Summary

Based on the local stress analyses, several notable problems can be found in the design. The most important problem is the abnormally high stress found in region C near the hole. This problem can most likely be solved by strengthening that region so that there is more material supporting against the thermal stresses and the helium pressures. There is also the question of how many helium support rails should be present in region C and how closely they should be placed together. More and thicker rails will mean less stress but also will constrict flow and lower heat transfer efficiency.

Another important problem for future analysis is that most high stress zones for each region appear at the corners and edges of the various regions. The borders between regions need more thorough analyses before we can conclusively say how high the stresses in those regions reach.

Finally, the problem with the inner corners (called re-entrant corners in Pro/M) cannot be solved with regular FEM method. In order to accurately gage the stresses in those regions, we need a more powerful method or physical testing, which is more preferable.

All of the analyses completed in this study assumed the worst possible steady-state conditions in a normal operational IHX. Real steady-state stresses will most likely be lower than what is given, while transient thermal stresses may be higher. It should be noted that even the best, most accurate set of constraints used to do stress analyses will produce some artificial (fake) stresses simply because the real part can move freely. Which is why making a real prototype and testing it is very important.

5. RECOMMENDATIONS AND FURTHER ANALYSES

The high temperature heat exchanger plates need to be redesigned based on the results of the above analyses. Problems like abnormally high stresses in region C need to be tackled first. Then subsequent optimizations of the boundaries between regions and the geometries of each region can be done to further lower the stresses present. Also, the analyses were based on simple linear temperature distribution and are likely to be inaccurate in some parts. Based on how large thermal effects can be on a global scale, one should find a better approximation for the temperature distribution. This may require the use of advance computation techniques. Finally, a scaled prototype should be constructed and tested for actual stresses present in the heat exchanger.

REFERENCES


APPENDIX A: APPROXIMATIONS IN UNIT CELL MODELS

Unit Cell B

Figure A1 shows the model for unit cell B. Unit cell B is not strictly a symmetric unit cell. It is a periodic unit cell, which means that multiple copies of this unit cell stacked in all directions can reproduce the overall structure of region B. In region B, the helium guide rails and the molten salt channels run at an angle to each other. The closest model we can make to a proper symmetric cell is to use periodic boundaries instead of mirror boundaries. However, it will still give fairly good results because if we can sub-divide the region into repeating cells, then all the those cells in this region will experience the same stresses when under the same pressure. This guarantees that analyzing only one of these repeating cells will give the stresses for the entire region, and that approximating this region with a solid cell of the right moduli is a pretty accurate approach. As with unit cell A, unit cell B has rounds at the roots of its fins but not at the top. These features are found in unit cell C and D as well. Because the roots represent a relatively small mass, we can ignore the asymmetric effect.

Figure A1: Unit cell B.
Unit Cell C

Figure A2 shows the model for unit cell C. Unit cell C is very similar to unit cell B except in region C the helium guide rail runs perpendicular to the molten salt channels. This geometry is very easy to model in comparison to unit cell B. Four molten salt channels were chosen so that the length of the cell in the z-direction is about equal to the length of the cell in the x-direction. A squarish shape is advantageous for the purpose of shear moduli test since you could run multiple shear tests on the same model.

![Figure A2: Unit cell C.](image)

Unit Cell D

Figure A3 shows the model for unit cell D. In region D, the helium plate is completely solid so it is best to build an approximately cubic cell. In this region, the molten salt channels are closer together than in previous unit cells because those channels are converging on the inlet/outlet. This region is very important not only because of the high stresses in the proximity of the hole but also because having salt channels packed close together means less solid material to support against stress. Besides ignoring the asymmetric effect of the rounds at the roots of the salt channels, no additional approximations were made.
APPENDIX B: DERIVATION OF EQUATIONS

To get the equations for finding the normal stress moduli, we begin by writing out parts of Eq. 1, the stress-strain relationship with the inverse elastic tensor, in its matrix form.

\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33}
\end{bmatrix}
= \begin{bmatrix}
i_{11} & i_{12} & i_{13} \\
i_{21} & i_{22} & i_{23} \\
i_{31} & i_{32} & i_{33}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33}
\end{bmatrix}
\]

(B1)

Now we perform three tests called A, B, and C. In each test we use an independent \( \varepsilon \) vector. The \( \sigma \) vector that is outputted by Pro/M will be recorded. For convenience, the \( \varepsilon \) values from test A will be called \( \varepsilon_{A1} \), \( \varepsilon_{A2} \), and \( \varepsilon_{A3} \), and the \( \sigma \) values will be called \( \sigma_{A1} \), \( \sigma_{A2} \), and \( \sigma_{A3} \). The values from tests B and C will be similarly named. By substituting values from the three tests into Eq. B1 and writing out the matrix multiplication, we can derive the following nine equations.

\[
\varepsilon_{A1} = \sigma_{A1}i_{11} + \sigma_{A2}i_{12} + \sigma_{A3}i_{13}
\]

(B2)

\[
\varepsilon_{A2} = \sigma_{A1}i_{21} + \sigma_{A2}i_{22} + \sigma_{A3}i_{23}
\]

(B3)

\[
\varepsilon_{A3} = \sigma_{A1}i_{31} + \sigma_{A2}i_{32} + \sigma_{A3}i_{33}
\]

(B4)

\[
\varepsilon_{B1} = \sigma_{B1}i_{11} + \sigma_{B2}i_{12} + \sigma_{B3}i_{13}
\]

(B5)

\[
\varepsilon_{B2} = \sigma_{B1}i_{21} + \sigma_{B2}i_{22} + \sigma_{B3}i_{23}
\]

(B6)

\[
\varepsilon_{B3} = \sigma_{B1}i_{31} + \sigma_{B2}i_{32} + \sigma_{B3}i_{33}
\]

(B7)
\[ \varepsilon_{C1} = \sigma_{C1}i_{11} + \sigma_{C2}i_{12} + \sigma_{C3}i_{13} \]  
(B8)

\[ \varepsilon_{C2} = \sigma_{C1}i_{21} + \sigma_{C2}i_{22} + \sigma_{C3}i_{23} \]  
(B9)

\[ \varepsilon_{C3} = \sigma_{C1}i_{31} + \sigma_{C2}i_{32} + \sigma_{C3}i_{33} \]  
(B10)

There are nine equations and nine unknowns, so you can solve for all the moduli values, i’s. For example, if you take Eqs. B2, B5, and B8, and rewrite them on together, you get a simple matrix equation that can be solved by calculators easily.

\[
\begin{bmatrix}
\varepsilon_{A1} \\
\varepsilon_{B1} \\
\varepsilon_{C1}
\end{bmatrix}
= 
\begin{bmatrix}
\sigma_{A1} & \sigma_{A2} & \sigma_{A3} \\
\sigma_{B1} & \sigma_{B2} & \sigma_{B3} \\
\sigma_{C1} & \sigma_{C2} & \sigma_{C3}
\end{bmatrix}
\begin{bmatrix}
i_{11} \\
i_{12} \\
i_{13}
\end{bmatrix}
\]  
(B11)

A simple inverse multiplication will yield the moduli \( i_{11}, i_{12}, \) and \( i_{13} \). The other six moduli can be found in a similar way.

**APPENDIX C: ADDITIONAL LOCAL TEST RESULTS**

In this appendix, additional results from the local stress tests have been included. The following four figures show the max/min stresses in unit cell B and C under the worst case scenarios. These results have been separated from the main body of the report because their accuracies are in doubt. They represent a very small minority of their respective region that also happens to be close to the boundaries of the region. Stresses found at these areas are inaccurate because interfaces contain complex geometric structures.

The stresses in unit cells B and C can be quite high, especially in unit cell C. However, it should be noted that the conditions in Figure C3 and C4 occur at a very small corner in the interface between region C and region O. Separate tests on these regions must be done to see if the stresses are truly this high.

Additionally, the stresses seem to be primarily supported by the helium guide rails. The areas around the guide rails show very high concentration of stresses. As mentioned in the main body of the report, this brings up the question of how thick the guide rails should be. If the guide rails are too thick, heat transfer efficiency will be compromised.
Figure C1: Max principle stress for Cell B at 3 MPa tensile stress (scale: 30 to 110 MPa).

Figure C2: Min principle stress for Cell B at 25 MPa pressure (scale: -30 to -150 MPa).
Figure C3: Max principle stress for Cell C at 50 MPa tensile stress (scale: 100 to 900 MPa).

Figure C4: Min principle stress for Cell C at 35 MPa pressure (scale: -50 to -450 MPa).