Intermediate Heat Exchanger Dynamic Thermal Response Model

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Abstract

This report presents UCB progress in developing a comprehensive thermal and fluid dynamics model for the NGNP intermediate heat exchanger (IHX) and other compact heat exchangers. For nuclear hydrogen applications, an IHX is required to transfer heat from high temperature and high-pressure primary helium coolant to a hydrogen production process. An intermediate heat transfer loop is used for the purpose. Under these conditions, plate-type heat exchangers with small flow channels, such as the well known Heatric designs, are a major candidates because they can achieve high power densities with small amounts of material, and can be fabricated using a diffusion bonding process so that the entire heat exchanger has the strength of the base material. However, these types of heat exchangers can be susceptible to very large stresses during thermal transients, for example when the flow of one fluid is interrupted abruptly. UCB has proposed a capillary shell and tube IHX configuration that could have lower susceptibility to thermal shock. For all IHX options accurate analysis of global and local thermal stresses are critical to evaluating the heat exchanger reliability and safety.

In order to estimate the stresses in compact heat exchangers a comprehensive thermal and hydraulic model is needed. The model developed here uses an effective porous media (EPM) approach because the evaluation of the detailed global flow with computational fluid dynamics (CFD) as well as finite element methods (FEM) for the mechanical analysis, at the resolution scale of the flow channels involves prohibitive computational time. The EPM fluid dynamics and heat transfer computational code developed at UCB is called the compact heat exchanger thermal and hydraulics (CHEETAH) code. CHEETAH solves for the transient temperature-distribution in the IHX. This temperature distribution can then be imported into a commercial finite element analysis (FEA) code for mechanical stress analysis using the EPM methods developed earlier by UCB for global and local stress analysis [2]. These simulation tools will also allow the designer to optimize the heat exchanger design, to minimize the pressure drop while maximizing the IHX’s thermal effectiveness, as well as to optimize the mechanical performance of the IHX particularly as it relates to creep deformation and transient thermal stresses.
Contents

Introduction........................................................................................................................................3
Media Description..............................................................................................................................5
Zoning the IHX ...................................................................................................................................6
  Diffuser and Reducer Permeability ...............................................................................................7
  Adjustable Flow Distribution .........................................................................................................8
  Specifying the Grid .........................................................................................................................9
  Rectangular Grid ..............................................................................................................................9
  Staggered Grid ...............................................................................................................................10
Fluid Dynamics ..................................................................................................................................10
  Fluid Dynamics Equations .............................................................................................................12
Effective Permeability ......................................................................................................................17
Heat Transfer ......................................................................................................................................19
  Heat Transfer Discretization ..........................................................................................................21
Preliminary Heat Transfer Results and Conclusions ........................................................................22
Future Work .......................................................................................................................................24
References ..........................................................................................................................................24
Introduction

The production of hydrogen using high temperature reactors requires intermediate heat exchangers to transfer heat from a primary coolant to an intermediate coolant. The intermediate liquid salt or gas loop acts as a buffer between the nuclear reactor and the hydrogen or chemical plant. This is intrinsically beneficial for overall system safety because by increasing the thermal inertia in the system (especially with LS) the intermediate loop also helps reduce the sensitivity to temperature transients.

An IHX is used to remove heat from the reactor-side of the intermediate loop and a process heat exchanger (PHX) is used to remove the high temperature heat from the intermediate loop. This high quality heat can then be used to produce hydrogen though chemical processes that dissociate water, or to heat steam for high temperature electrolysis. Figure 1 shows a typical Heatric type heat exchanger. Figure 2 shows a schematic of a capillary tube and shell heat exchanger concept developed at UCB intended to reduce the effects of transient thermal stresses [10].

Figure 1: Photo of a cutaway-model of a typical Heatric heat exchanger showing multiple inlet and outlet manifolds and slices across various plates and flow channels.
The temperature input into the power plant occurs at the IHX, which makes this temperature critical to the plant’s thermodynamic efficiency. Still, the heat exchanger must be able to withstand the rapid temperature changes that can result during a reactor shutdown or interruption of the intermediate coolant flow, as when one of the fluid circulators or pumps suddenly trips off. In scenarios such as these, the thicker sections away from the enhanced heat transfer region (fins and flow channels) will take longer to thermally equilibrate than the thinner heat transfer surfaces. That transition period is critical to the integrity of the IHX because large thermal gradients will occur in areas that also may have internal features that act as stress concentrators. For an offset strip fin HX design,

![Diagram of a capillary tube and shell heat exchanger](image)

**Figure 1:** Capillary tube and shell heat exchanger showing the proposed tube-bundle geometry formed by diffusion bonding of multiple bundles of ~2,500 3.0-mm diameter tubes with hexagonally tapered ends to form inlet and outlet tube sheets [10].

It is evident from some of the sharp angles near the manifolds in Fig. 1 that the relatively elaborate geometry in the manifold region can lead to a very complex stress distribution over the IHX that will vary in time as the different sections thermally equilibrate during a thermal transient. Again, attempting to model this fluid, temperature and stress distribution directly at the channel level for the entire heat exchanger is computationally prohibitive because of the different length scales that must be resolved.
Still, it is critical to be able to predict approximate stresses over the entire IHX (e.g., a global model). This requirement led the current study towards the EPM approach.

Previous work at UC Berkeley [2] developed an alternative method to obtain approximate stresses that greatly reduces computational effort. This method is composed of three steps. First, the heat exchanger is broken down into several regions. Unit cell models are built based on each region that captures all of the most important features of that region. The effective mechanical and thermal properties for each unit cell are then determined using FEM simulations. Second, global stress distributions based on the volume-averaged unit-cell-equivalent model are computed by using the effective mechanical and thermal properties. Third, the values from the stress distributions are then applied to the unit cells to find localized points of high stress.

The fluid dynamics component of the model was first developed for the offset strip fin (OSF) region in only one dimension and with a constant permeability. The model can also apply to other flow channel geometries. Later, spatially varying permeabilities were included as well as areas with no permeability to represent no flow boundaries and the solid portions of the IHX. Analysis for both high-pressure helium and liquid salts has been performed. The small characteristic length (hydraulic diameter) of the region and in the manifolds in addition to the creeping flow of the liquid salt produces a Darcian flow regime in the liquid salt plate. Furthermore, the incompressibility of the liquid salt greatly simplifies the fluid mechanics. This is less so the case on the helium side of the IHX where the flow rates are much higher and where the compressibility of the gas could play a small role in the transient fluid dynamics. Established friction factor correlations can be manipulated to produce an effective permeability (which also serves as a correction factor as the fluid dynamics deviate from the Darcian regime) and Darcy’s transport equation again relates the fluid velocity to the pressure drop. This is discussed in more detail in the fluid mechanics section of this report.

The heat transfer part of the problem has been developed progressively from one dimensional models that neglected longitudinal (main flow direction [x]) conduction to two dimensional models that include cross flow in the fluids as well as spatially varying longitudinal and latitudinal (cross flow direction [z]) conduction. Similarly, early models used constant convection coefficients and porosity while the more recent version includes spatially varying convection coefficients and porosity. Eventually, the CHEETAH model may accommodate convection coefficients that vary with the local fluid velocity.

**Media Description**

The IHX can be made of one of several high temperature metallic alloys the way current Heatric HX are currently manufactured but UCB has also done extensive work to explore the possibility of making a ceramic IHX through a molding process. While the ceramic IHX would have a lower conductivity and would be less compliant under thermal stresses, it would able to operate at much higher temperatures without creep deformation, increasing plant efficiency and producing heat at temperatures necessary for
chemical hydrogen production. These silicon-carbide based ceramic materials are also the primary candidates for use in the process heat exchangers.

In modeling the IHX, the actual material properties can be set and changed with great ease since they are simply represented by a series of constants assigned in the input file of the code. However, the fluidic, thermal and mechanical response of the solid media comes from geometric effects that are central to the model’s validity. This is where the EPM model simplifies the problem greatly by creating mass and energy balances at a scale that will both represent the complexity of the thermal hydraulics in the IHX and yet produce simulation results in a time period that allows researchers to improve the IHX design from a fluidic, thermal and mechanical perspective through several design iterations and computational simulations. Calculating representative constants that correspond to effective x-dir conductivity, z-dir conductivity, x-dir permeability, z-dir permeability, porosity, convection coefficients for LS and/or He plates, elastic modulii, shear modulii & Poisson’s ratios is very labor intensive because it takes over 20 unique zones with different media properties to fully describe the hydraulic, thermal and mechanical properties of a typical compact heat exchanger. This is done with various scripts within the CHEETAH code.

**Zoning the IHX**

The assignment of constants to the property matrices in the CHEETAH code is a cumbersome but essential part of the EPM model. One central purpose of the CHEETAH code is to enable the designer to simulate the performance of the IHX with a given geometry and set of dimensions so that the exchanger design can be improved in an iterative fashion based on the simulation’s results. This means that the IHX’s dimensions are very much subject to change while the general shape changes little.

With this in mind, the CHEETAH code is designed with a long set of system parameters that make changing the dimensions on the IHX model as easy as changing the values of a few constants that represent those dimensions and system parameters. Then, the CHEETAH code will automatically change the zone boundaries so that the user doesn’t have to modify the code with every dimensional variation. Modification of the code may be necessary if any of the dimensions approach the grid size or if the shape of the IHX is modified. A schematic drawing of the IHX’s porosity distribution is included in Figure 3 to illustrate some of these zones with different colors.
Figure 3: A schematic of the IHX’s porosity distribution by zones

For example, the zones shown in purple in Figure 3, above, represent a porosity of zero ($\phi = 0$) which means that these zones will have a larger thermal diffusion length scale and all else being equal will take longer to equilibrate during thermal transients.

**Diffuser and Reducer Permeability**

The schematic drawing above also is simplified in the diffuser and reducer areas (trapazoids shown in orange). In these zones every control volume has unique effective properties assigned to it depending on the local channel geometry being represented by the EPM. In order to accurately represent the local geometry the assigned EPM uses the angle and spacing of the channels nearest the node in the actual IHX. The CHEETAH code accomplishes this by actually finding the position of each node and the slope of the line connecting the node point to a central fictitious point to which the flow distribution channels align. Using the point and slope information for each node and the line equation from adjacent channels the CHEETAH code finds the normal distance between channels at that node point and uses this to calculate a multiplier for the directional permeability so that the effective x and z-dir permeabilities at that node take into consideration not only the alignment of the channel but also the local channel density. These zones are shown in a uniform color here for simplicity but each is also includes other parameters that allow the CHEETAH code user to manipulate the flow distribution. This will be discussed further in the next section. Schematic drawings similar to Fig. 3 would illustrate the layout of other effective property values such as x-dir conductivity, z-dir conductivity, x-dir permeability, z-dir permeability, porosity, convection coefficients for LS and He, elastic modulii, shear modulii and Poisson’s ratios the IHX.
Adjustable Flow Distribution

In addition to accommodating the channel orientation and channel density in the nodal permeability assignment the CHEETAH code also allows the designer to manually tune a multiplier coefficient for distinct zones in the manifold sections. The manifolds play a key role in creating uniform flow in the OSF region which in turn is crucial to maximizing the effectiveness of the IHX. The adjustable permeability in these zones allows the user to quickly and directly manipulate the flow distribution in the OSF region as if controlling independent and parallel valves leading into and out of the OSF section. Shown in yellow, orange, pink, red and maroon, these flow regions can be seen in the inlet and outlet manifolds shown below of the liquid plate schematic drawing in Fig. 4.

![Diagram of adjustable flow distribution](image)

**Figure 4: Independent flow regions in the inlet and outlet manifolds**

The lower flow resistance associated with the shortest flow path in the inlet manifold is already balanced by a higher flow resistance created by the corresponding longer flow path in the outlet manifold so that the total flow resistance is roughly equal regardless of the path followed. However, there are still issues with flow maldistribution in the IHX. Among the factors that affect the flow distribution are the temperature dependant viscosity effects (not yet incorporated into CHEETAH), longer flow paths associated with the diffuser and reducer path as well as the fact that the paths nearest the inner corners of each region are shortest and therefore see higher flow rates than the outside corners in each flow region.

Each manifold (which includes the diffuser and reducer section) is separated into several flow regions. The number of flow regions is controlled by the assignment of a constant (chanmax) in the code. The manifolds can be divided into any number of flow regions but it is important to maintain a region width of at least 3 cells across to avoid discretization errors such as “staircasing”, where the coarse mesh creates problems of
unintended reductions in flow area and therefore peak velocities that can lead to instabilities by way of excessive CFL (Courant – Friedrichs – Levy = \( u^*\Delta t/\Delta x \)) numbers.

**Specifying the Grid**

The CHEETAH Code uses a rectangular mesh which fits the rectangular geometry of the IHX and with the rectangular matrices produced in MatLab. Originally the code was written with a typical rectangular grid such that each node point lies in the center of the control surface and was identified by four coordinates \((x – \text{position}, y – \text{position}, z – \text{position}, \text{iteration number})\) with the following indexes \((i,j,k,n)\). The y-position (“j” index) in the rectangular grid connotes the phase with index 1 and 3 being the fluids and 2 always representing the solid phase. The CHEETAH Code was later improved by replacing the simple rectangular grid with a more sophisticated staggered grid.

The \(x\) and \(z\) – position coordinates associated with every node when multiplied by the corresponding spatial discretization \(\delta x \& \delta z\), respectively, provide a position for the node point in the IHX geometry. That position then led to the assignment of the corresponding local physical properties such as porosity, \(x\) & \(z\) permeability, \(x\) & \(z\) conductivity, convection coefficients, pressure and temperature according to the zoning described previously.

**Rectangular Grid**

In this method of discretization all properties are assigned to the center of the control volume and mass conservation is checked at \((i,k)\) by applying the following equation:

\[
 u(i-1,k) - u(i+1,k) + w(i,k-1) - w(i,k+1) = 0
\]

While simple, this type of discretization actually uses a control volume 4 times larger than the actual grid spacing. This is because for continuity the velocities must be summed at the boundary of each cell but since the velocities are defined at the center of each cell the code must calculate velocities from neighboring cells or must find an average velocity between to cells to approximate a boundary velocity. If neighboring cell velocities are used directly then this creates a much coarser grid than intended and significantly diminishes the accuracy of the results. If an average velocity is found then the computation time is increased. Furthermore, this method can also lend itself to “zigzag” or “checkerboard” pressure or temperature distributions since the continuity and momentum equations use information from alternating nodes to check for continuity. Essentially, this means that a wavy pressure distribution will produce a wavy velocity distribution that can satisfy continuity since it is information from alternating and not adjacent nodes that must satisfy continuity [4].
**Staggered Grid**

An alternative to the rectangular grid is the staggered grid. This method involves using several matrices to describe the physical properties of the control volumes. Those properties that are defined at the center of the control volume (blue nodes) such as temperature, pressure, and porosity are still defined there, as before. However, those properties that are best defined at the left and right boundaries such as x-permeabilities, x-conductivities and x-velocities are defined on a new matrix that is shifted to the left by half grid space (red nodes) so that they lie right on the x boundary.

The same is true for the properties that must be accounted for on the upper and lower boundaries of the control volume. This refers to the z-permeabilities, z-conductivities and z-velocities which are defined on a new matrix that is shifted to the down by half-a-grid space (green nodes) so that they lie right on the x boundary.

By using the staggered grid, the code is able to incorporate a much finer mesh and is able to ensure continuity at every control volume using velocity components from adjacent nodes. Also, the pressure drop between adjacent cells becomes the natural driving force for the velocity that is calculated between them at the face of the control volume of the adjacent cell [4].

**Figure 6: Staggered Grid in x-dir**

**Figure 7: Staggered Grid in x & z-dir**

**Fluid Dynamics**

The fluid mechanics in the IHX are solved for both fluid phases by breaking up the IHX into over 20 flow zones. Each of these zones has unique physical properties that govern the flow for each of the two phases. For the case of an IHX that transfers heat
between a liquid and a gas, it is representative to take the case of liquid salt and helium. Figures 8 and 9 show the zoning for these two plates, respectively.

**Figure. 8:** Schematic of the IHX section for a LS plate

**Figure. 9:** Schematic of the IHX section for a He plate

The IHX is built from a diffusion bonded stack of alternating gas and liquid (or gas) plates as shown in Figure 10. Due to the stacked assembly, the flow areas on the liquid and gas plate overlap all over the IHX and create zones in the solid material with complex thermal properties. The zones in the liquid plate are completely separate form
the zones in the gas plate, so the fact that the zones overlap has no effect on the fluid mechanics. The overlap intimately affects the heat transfer, however.

\[ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \]  

Equation 1

Then, it is known that for very small Reynolds’ number flows, Darcy’s transport equation yields excellent accuracy with a medium dependant permeability. For laminar flows with a Reynolds’ number larger than 1 the permeability coefficient can be adjusted so that it becomes a function of the Reynolds’ number. This is done by solving for the Fanning friction factor, \( f_f \) and using this to find the effective permeability, \( k_x \) & \( k_z \).

Darcy’s transport equation for 1D flow in porous media is

\[ u = u_{int} \cdot \phi_z = -\frac{k_z \rho g \, dh}{\mu} \frac{dh}{dx} \quad \text{or} \quad u = -\frac{k_z}{\mu} \frac{d\Phi}{dx} \]  

Equation 2a

\[ w = w_{int} \cdot \phi_x = -\frac{k_x \rho g \, dh}{\mu} \frac{dh}{dz} \quad \text{or} \quad w = -\frac{k_x}{\mu} \frac{d\Phi}{dz} \]  

Equation 2b

Figure 10: Cut away view through the OSF section showing alternating liquid salt (top and bottom arrows) and helium (middle arrows) flow channels. Dark bands at the top of each fin indicate the location of joints between each plate [1].

Fluid Dynamics Equations

Beginning with the equation of continuity it is known that mass must be conserved in any representative control volume in the field of flow such that:
Here, \( \Phi \) is the flow potential [Pa] and \( u \) is the Darcy velocity.

The following section will describe how to obtain the effective permeability from correlations for the Fanning friction factor and also will detail some of the advantages in using this approach. In order to find the velocity distribution in the IHX it is necessary to first find the pressure distribution. The following equation shows one way to accomplish this.

The partial derivative of the x component of the fluid velocity with spatially varying permeability \( k \), and dynamic viscosity \( \mu \), gives due to the spatially varying viscosity and permeability:

\[
\frac{\partial u}{\partial x} = -\frac{1}{\mu} \frac{\partial k}{\partial x} \frac{\partial \Phi}{\partial x} + \frac{k}{\mu^2} \frac{\partial \mu}{\partial x} \frac{\partial \Phi}{\partial x} - \frac{k}{\mu} \frac{\partial^2 \Phi}{\partial x^2} \quad \text{Equation 3}
\]

Combining equations 1 & 3 gives the following elliptic differential equation:

\[
-\frac{1}{\mu} \frac{\partial k}{\partial x} \frac{\partial \Phi}{\partial x} + \frac{k}{\mu^2} \frac{\partial \mu}{\partial x} \frac{\partial \Phi}{\partial x} - \frac{k}{\mu} \frac{\partial^2 \Phi}{\partial x^2} + \frac{1}{\mu} \frac{\partial k}{\partial z} \frac{\partial \Phi}{\partial z} + \frac{k}{\mu^2} \frac{\partial \mu}{\partial z} \frac{\partial \Phi}{\partial z} - \frac{k}{\mu} \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad \text{Equation 4}
\]

Initially, this equation was solved only within certain zones with constant x & z permeability, under the assumption of constant fluid properties such as a constant dynamic viscosity, \( \mu \). For this case the Equation 4 is reduced to the following elliptic equation:

\[
\frac{k_x}{\mu} \frac{\partial^2 \Phi}{\partial x^2} + \frac{k_z}{\mu} \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad \text{Equation 5}
\]

This equation was solved with appropriate boundary conditions using finite differencing with Taylor-series approximations for each phase and it provided the steady state pressure fields for each fluid. This pressure field readily gave the velocity field for the areas of constant properties. However, while this set of assumptions worked well within the OSF portion of the IHX due to its constant permeability, the assumptions were clearly invalid in the inlet and outlet manifolds where the geometry changes frequently and abruptly.

In order to estimate the velocity field in these areas the more complicated Equation 4, above, had to be solved. However, when it too was approached using the finite difference scheme with Taylor-series approximations, large instabilities resulted in the zones where the permeability varies from node to node such as in the diffuser, reducer and also at the interface between two zones of different permeability such as between the OSF region and the inlet and outlet manifolds. It seemed the instabilities were coming from the first derivatives of permeability with respect to space, \( \frac{\partial k}{\partial x} \) and \( \frac{\partial k}{\partial z} \).

\textit{Fluid Dynamics Discretization & Results}
This obstacle was overcome by approaching the problem with finite volume analysis (FVA) or control volume approach. In this approach the conservation equation is applied to a discrete area, volume or grid from the beginning, see Figure 6 below. The velocities come from the pressure relationships given in Darcy’s transport equation in Equation 2, above. Unlike the Taylor-series based approaches from before, the FVA does not take the limit by reducing the control volume down to a point. This provides a significantly more stable solution since the conservation equation is asserted on each discrete area or volume in the grid [6].

\[ w(i,j,k+1) \]
\[ u(i,j,k) \]
\[ \Phi(i,j,k) \]
\[ u(i+1,j,k) \]
\[ w(i,j,k) \]

**Figure 11:** Finite volume analysis or control volume approach for fluid dynamics solution.

An iterative FVA technique was used to solve for the pressure distribution in the liquid salt plate. Figure 12 shows the resulting steady state pressure distribution in the IHX.
Figure 12: Liquid salt plate pressure distribution solved with the finite volume analysis or control volume approach.

Clearly the pressure gradient varies greatly in the diffuser and reducer, as expected. Also the gradient appears flat in the core of the IHX where the permeabilities are constant. Sharp changes in the pressure gradient can be seen where the OSF sections meet the inlet and outlet manifold. This identifies the interface without the instabilities seen with the Taylor-series based finite difference techniques. Outside the flow zones, the pressure gradient is flat and the pressure is set at a lower constant to indicate that this is not a flow region.

Using the pressure distribution solved above and the permeability matrix assigned previously, Equation 2 can give a velocity field in the x and z-direction. Summing the x and z-direction at every mesh provides a non-directional total flow distribution in the liquid plate of the IHX. This flow field can be seen in Figure 13.
Figure 13: Liquid salt plate flow distribution solved with the finite volume analysis.

Notice the increased flow rates in the diffuser and reducer (at the inlet and outlet). As expected in the diffuser (right side of Figure 13) as the cross section increases the flow rate drops sharply and the flow rate accelerates as the cross section is reduced in the nozzle-like reducer (left side of Figure 13). The same effect can be observed with the rise in the flow rate in the z-dir oriented flow channels of the inlet and outlet manifold. Furthermore, at the entrance to the OSF region the liquid can be seen to increase in velocity close to the near-side corner of each flow section, as it flows preferentially through the path of least resistance (in this case the shortest path being just around the nearest the corner).

Following the same procedure for the helium plate Figure 14, below, shows the steady state pressure distribution for the helium plate in the IHX.
Figure 14: Helium plate pressure distribution solved with finite volume analysis.

The simple fluid dynamics of the helium plate are evident in the above pressure distribution where the permeabilities in the helium-side OSF region are constants in both x and z directions.

**Effective Permeability**

For this model it is important to show that the flow has reached a hydrodynamically fully developed condition. The offset fin studies of Sparrow et al. and Kelkar and Patankar show that for Graetz number, Gz< 200 the flow has effectively reached a periodic hydrodynamics fully developed condition [7].

\[
Gz = \frac{Re^* D_z}{x} \quad \text{Equation 6}
\]

For the IHX the hydrodynamic entry length is a fraction of a percent of the length of the core section.

For 1D fully developed laminar flow in a pipe the Navier-Stokes equations of motion reduce to:
\[ \bar{u} = \frac{-D^2}{32\mu} \frac{d\Phi}{dx} \text{ or } \bar{u}^2 = \bar{u} \ast \frac{-D^2}{32\mu} \frac{d\Phi}{dx} = -\text{Re} \frac{D}{32\rho} \frac{d\Phi}{dx} \]  

Equation 7

Here it can be clearly seen that \( \frac{D^2}{32} \) serves as the geometry dependant correction factor similar to the permeability which has the same units.

The Fanning friction factor for laminar flow in a pipe is

\[ f_f = \frac{16}{\text{Re}} \]  

Equation 8

Combining Equations 7 & 8 yields

\[ \bar{u}^2 = -\frac{D}{2f_f\rho} \frac{d\Phi}{dx} \]  

Equation 9

and dividing through by the average fluid velocity gives the following equation for flow in a pipe.

\[ \bar{u} = -\frac{D\mu}{2f_f\rho \bar{u} \mu} \frac{1}{dx} \]  

Equation 10

Applying this to the effective porous media which also concerns laminar fully developed flow and applying the hydraulic diameter \( D \) becomes \( D_h \) and \( \bar{u} \) (average velocity in a pipe or analogous to the interstitial velocity in the effective porous medium). Then \( U_D \) (Darcy velocity) divided by the directional porosity is equivalent to the interstitial velocity, as follows:

\[ \bar{u} = U_{\text{int}} = \frac{U_D}{\phi_x} \]  

Equation 11

This in turn allows Equation 10 to be re-written as

\[ U_D = -\frac{D_h\mu\phi_x^2}{2f_f\rho U_D \mu} \frac{1}{dx} \]  

Equation 12

This finally determines the relationship between the Fanning friction factor and the effective permeability as:

\[ k_x = \frac{D_h\mu\phi_x^2}{2f_f\rho U_D} \]  

Equation 13

This result plays a very important role in the implementation of the fluid dynamics of the CHEETAH Code. First, it allows the script to discretize Darcy’s transport equation in its simplest form as seen in Equation 2. It is important to note that the permeability is now a function of velocity, but it is equally important to note that the Fanning friction factor is a function of velocity as well through the Reynolds number. At very low flow rates (Re<1) the velocity dependence will be eliminated.
A real advantage of this technique is the fact that the fluid potential distribution is found with the same equation in core flow zones, the boundary conditions and the no-flow areas provided the permeability distribution is correctly specified. This also implies that the best correlations giving the Fanning friction factor for any complex geometry in laminar fully developed flow can be used to generate an effective permeability which will accommodate the flow rates in the range of validity of the Fanning friction factor correlation.

From a programming standpoint this makes the CHEETAH Code very versatile and easy to adjust for different fin arrangements and geometries or dimensions because only the Fanning friction factor correlation in the code must be changed. This is a very significant advantage versus the alternative of discretizing the correlation itself. If the correlation was discretized using the Fanning friction factor directly, while the solution would be the same, this would make changing the code to reflect dimensional or geometric changes very cumbersome because this momentum equation appears all over the code with a few adjustments for the boundaries and zones. In short, the effective porous media (EPM) treatment is very advantageous!

**Heat Transfer**

After the fluids module resolves the velocity distribution in the axial (x) and cross-flow (z) directions, the thermal module of the IHX code applies an energy balance using a similar finite volume analysis (FVA) or control volume analysis as discussed previously in the fluid mechanics sections. Here too, the differential equations stemming from the energy equation were initially solved using the FDM with Taylor-series approximations until the CHEETAH code encountered instabilities when modeling the consolidated IHX, which includes many zones with properties that vary spatially and abruptly. In fact, the differential equations below result from the assumption that only temperature varies spatially and that variables such as velocity, density, specific heat, conductivity, convection coefficient, and porosity are all spatially independent. Clearly, this is not the real case, but still the following equations are valid within the independent zones of the IHX where it is reasonable to assume the above mentioned properties as constant and the FDM was used to solve this system of equations before the development of the consolidated CHEETAH code. The equations are also included here because they illustrate the dominant physical phenomena that govern the energy transport in the IHX for each phase.

**Heat Transfer Equations**

**Hot Fluid (fh)**  

\[
-u_{fh}\rho_{fh}c_{ph} \frac{\partial T_{fh}}{\partial x} - w_{fh}\rho_{fh}c_{ph} \frac{dT_{fh}}{dz} + k_{fh}a'_{fh} \frac{\partial^2 T_{fh}}{\partial x^2} + k_{fh}a'_{fh} \frac{\partial^2 T_{fh}}{\partial z^2} - h_{fh}a'_{fh} (T_{fh} - T_s) = \phi_{fh}\rho_{fh}c_{ph} \frac{\partial T_{fh}}{\partial t}
\]
An important dimensionless number to consider here is the Peclet number, which provides the ratio of the advective energy transport (thermal energy transport due to mass transport) and the conductive thermal energy transport. The Peclet number is defined as:

$$Pe = \frac{u * \Delta x}{v} = \frac{u * \Delta x * \rho * c_p}{k}$$  \hspace{1cm} \text{Equation 15}$$

The Peclet number for the helium flow rate is roughly 7700. This clearly means that the heat is mostly due to mass transport while the conductive contribution is negligible.

Neglecting Conduction in the Hot Fluid the energy balance is reduced to:

$$-u_{h} \rho_{j} c_{ph} \frac{\partial T_{h}}{\partial x} - w_{h} \rho_{j} c_{ph} \frac{dT_{h}}{dz} - h_{ph} a_{ph}' (T_{h} - T_{s}) = \phi_{ph} \rho_{j} c_{ph} \frac{\partial T_{h}}{\partial t}$$  \hspace{1cm} \text{Equation 16}$$

Solid (s):

$$h_{ph} a_{ph}' (T_{h} - T_{s}) - h_{fc} a_{fc}' (T_{s} - T_{fc}) + k_{s} a_{s}' \frac{\partial^2 T_{s}}{\partial x^2} + k_{s} a_{s}' \frac{\partial^2 T_{s}}{\partial z^2} = \phi_{s} \rho_{s} c_{ps} \frac{\partial T_{s}}{\partial t}$$  \hspace{1cm} \text{Equation 17}$$

Cold Fluid (fc)  \hspace{1cm} \text{Equation 18}

$$-u_{fc} \rho_{fc} c_{pfc} \frac{\partial T_{fc}}{\partial x} - w_{fc} \rho_{fc} c_{pfc} \frac{dT_{fc}}{dz} + k_{fc} a_{fc}' \frac{\partial^2 T_{fc}}{\partial x^2} + k_{fc} a_{fc}' \frac{\partial^2 T_{fc}}{\partial z^2} + h_{fc} a_{fc}' (T_{s} - T_{fc}) = \phi_{fc} \rho_{fc} c_{pfc} \frac{\partial T_{fc}}{\partial t}$$

Again, the Peclet number for the liquid salt flow rate is roughly 7500, so the conduction effect is negligible.

Neglecting conduction in the cold fluid the energy balance is reduced to:

$$-u_{fc} \rho_{fc} c_{pfc} \frac{\partial T_{fc}}{\partial x} - w_{fc} \rho_{fc} c_{pfc} \frac{dT_{fc}}{dz} + h_{fc} a_{fc}' (T_{s} - T_{fc}) = \phi_{fc} \rho_{fc} c_{pfc} \frac{\partial T_{fc}}{\partial t}$$  \hspace{1cm} \text{Equation 19}$$

Lastly, it is important to clarify that the CHEETAH code solves the energy equations at every temperature node by summing advective contributions, for the fluid phases, and conductive contributions, for solid phase, at the boundaries of the control volume, $\Delta x \Delta y \Delta z$, just as was executed for the fluid mechanics portion. This is combined with convective heat transfer between the phases to give the increase or decrease in sensible thermal energy for each control volume which manifests itself as a change in temperature as illustrated in Figures 15 and 16

Pg. 20 of 24
Heat Transfer Discretization

Control Volume for Fluid Phases in IHX

\[
\begin{align*}
&u(i,j,k)q_cT(i-1,j,k) \\
&\phi q_c(T(i,j,k,n+1)-T(i,j,k,n)) / \Delta t \\
&w(i,j,k)q_cT(i,j,k) \\
\end{align*}
\]

Figures 15: Control volume for either the hot or cold fluid as approached via FVA

Control Volume for Fluid Phases in IHX

\[
\begin{align*}
&-ka'(T(i,j,k+1)-T(i,j,k)) / \Delta z \\
&ha'(T(i,j,k)-T(i,j,k+1)) \\
&-ka'(T(i,j,k)-T(i-1,j,k)) / \Delta x \\
&\phi q_c(T(i,j,k,n+1)-T(i,j,k,n)) / \Delta t \\
&ha'(T(i,j,k)-T(i,j-1,k)) \\
&-ka'(T(i,j,k)-T(i,j,k-1)) / \Delta z \\
\end{align*}
\]

Figure 16: Control volume for either the solid phase as approached via FVA
Preliminary Heat Transfer Results and Conclusions

The main concern with the IHX is the mechanical performance during rapid thermal transients. The CHEETAH code has been developed to model the transient temperature distribution during such events. In modeling this scenario, it is assumed that the power plant is operating at steady state at full power when a transient event takes place. Among other things, these scenarios may include an almost instantaneous tripping of one of the coolant pumps. From a thermal stress perspective the most likely, relatively severe thermal transient would involve either the primary or intermediate coolant pump tripping while the other continues to operate. The thermal response for this scenario was recently modeled by CHEETAH, and the results for both a liquid salt and helium pump trip are provided in Figure 17.

The bottom center of each graph in Fig. 17 shows the temperature outline from the diffuser in the inlet manifold of the liquid salt plate at the low inlet temperature. Just above it the reducer from the exit manifold of the liquid salt plate can be seen at a much higher outlet temperature. In this case the liquid salt is the cold fluid and it enters the IHX at 900K while helium is the hot fluid which enters the IHX at 1200K. For this reason, the vertical axis of the graphs spans from 850K to 1250K. In this study the OSF section of the IHX is shown to be 1 meter long with an inlet and outlet manifold which are each 60cm long. The IHX is 57cm wide (z-dir) while the diffuser and reducer only stretch 6cm from the opening. While this is a short distance for the diffuser and reducer, it also makes the bounding wall along the length of the OSF region 6 cm thick. This is the region that also appears in purple along the OSF region in Figure 3. This creates a relatively large thermal diffusion length scale, which is evident in observing the delay in thermal response in this region in Figure 17. Even after 50 seconds those bounding walls have not equilibrated with the OSF section and that delay causes very large thermal gradients early in the temperature transients.

This observation comes as a direct result of graphics and animations produced from the CHEETAH code and the information learned from the results that it provides incentive to refocus the attention of the IHX designer towards the most vulnerable areas of the design. Through an iterative process such as this one the IHX designers can quickly optimize the design and update CHEETAH to reflect the new dimensions or shape of the IHX. Eventually, when no obvious problems arise, the temperature distribution can be exported to a mechanical FEA software (likely ANSYS) for detailed thermal/mechanical stress analysis using effective mechanical properties to calculate strains and to then use these strains to back calculate the local stresses as demonstrated in previous work using effective mechanical properties at UCB [1].
Figure 17: Transient temperature response to interruption of flow of liquid salt (left) or helium (right)
Future Work

- Calculation of effective thermal and mechanical properties
- Include the welded pipe section on the IHX
  - This could prove to be a critical area
- Reduction of OSF bounding wall thickness to reduce thermal inertia
- Include temperature dependent Fluid Properties
- Include convection coefficients that vary with fluid velocity

References


